A Stochastic Process to Model the Fluctuations of Asphalt Cement Price

Mohammad ILBEIGI¹, Baabak ASHURI² and Yang HUI³

¹ Graduate Research Assistant, Economics of the Sustainable Built Environment (ESBE) Lab, School of Building Construction, Georgia Institute of Technology, Atlanta, Georgia. email: ilbeigi@gatech.edu
² Director, Economics of the Sustainable Built Environment (ESBE) Lab, Assistant Professor and Chair Integrated Project Delivery Systems, School of Building Construction Georgia Institute of Technology, Atlanta, Georgia. email: baabak.ashuri@coa.gatech.edu
³ Master of Science Student, School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, Georgia. email: yhui6@gatech.edu

ABSTRACT

Transportation agencies and highway contractors are facing great challenges of variations in construction costs for transportation projects. Significant volatility in costs of construction materials, such as asphalt cement, is one of the most important drivers of uncertainty about transportation project costs. This type of uncertainty leads to serious difficulties for contractors to estimate project costs accurately. However, there is little knowledge about how asphalt cement price fluctuates over time. This gap in knowledge makes it difficult for contractors and transportation agencies to estimate project cost accurately. The research objective of this paper is to model fluctuations of asphalt cement price by using an appropriate stochastic process. It is concluded that the Geometric Brownian Motion (GBM) is a good stochastic process to model random variations of asphalt cement price over time. A probabilistic approach based on the Monte-Carlo simulation is applied on the GBM model to simulate future random paths for asphalt cement price index. It is expected that this work helps contractors and transportation agencies systematically analyze variations in the price of asphalt cement and develop more accurate estimations for their transportation projects.

INTRODUCTION

Significant growth in the costs of commodity inputs is the major driver of highway construction costs (FHWA 2007). In particular, sharp increases in the price of asphalt cement, which is an essential material for transportation projects, is often argued as a major reason for increasing highway construction costs (Zhou and Damnjanovic 2011; Skolnic 2011; Damnjanovic and Zhou 2009; Gallagher and Riggs 2006; Wilmot and Cheng 2003). This increase in the cost of asphalt cement is due to increased crude oil prices which may fluctuate significantly in a short period of time. Therefore, the cost of asphalt cement has a significant impact on bidding prices and consequently project costs (Wang and Liu 2012).

Although price of asphalt cement increase over the long term, it is subject to considerable short-term variations. Volatility in the price can lead to serious problems for both owner organizations and contractors. Owner organizations may face
problems such as price speculation, hidden price contingencies or inflated bids, very short-term price guarantees and not enough bidders for a project. Also, contractors have severe problems for accurate estimation. They may lose their bids due to cost overestimation or lose their profits due to cost underestimation (Ashuri and Lu 2010).

The ability to model trends and variations in asphalt cement price can help owner organization and contractors to handle the risk of price volatility appropriately. However, there is little knowledge about how asphalt cement price fluctuates over time. This gap in knowledge makes it difficult for both owner organizations and contractors to develop proper risk management strategies for material price volatility. The research objective of this paper is to model fluctuations of asphalt cement using an appropriate stochastic process. It is expected that this work helps contractors and transportation agencies systematically analyze variations in the price of asphalt cement and consequently develop more accurate estimations for their transportation projects.

DATA SET: ASPHALT CEMENT PRICE INDEX

Liquid asphalt cement is one of the most critical materials in highway construction projects. During last decade the price of Asphalt Cement has had a considerable fluctuation. In the state of Georgia, monthly asphalt cement index is calculated based on the average price of the material from approximately 15 different suppliers in the state or in the neighboring states which can provide asphalt cement for projects in Georgia after removing the minimum and the maximum of the prices. Figure 1 shows the value of asphalt cement index in Georgia from October 2005 to June 2012 indicating high volatility in the price even over a short period of time.

Overall, as the figure shows, there is an upward trend during this period. However, the graph shows considerable fluctuations. Asphalt cement index hits a peak in September 2008 and then decreases drastically until May 2009. This huge fluctuation can be a disaster for contractors to forecast and manage their costs. Consequently, contractors try to consider risk premium in their bids to protect them from an unexpected increase in the prices.
STOCHASTIC PROCESS: GEOMETRIC BROWNIAN MOTION

In contrary to a deterministic model which predicts a single outcome based on a given set of conditions, a stochastic model predicts a set of possible outcomes based on their likelihoods or probabilities. A stochastic process is a group of random variables demonstrating the development of some random values or systems over time (Karlin and Taylor 1996). A Markov process is a particular type of stochastic process in which only the present value of a variable is important to predict the future.

A Wiener process or standard Brownian motion is a specific type of Markov process in which the mean change in the value of the variable is zero with the variance of change equal to one per unit time (Marathe and Ryan 2005). Therefore, a standard Brownian motion has two major characteristics. First, the change in the value of the variable follows a normal distribution with mean 0 and variance equal to the change in time. Second, the changes in the value of the variable for any two non-overlapping intervals of time are independent (Marathe and Ryan 2005). The standard Brownian motion process has a drift rate of zero and a variance of one indicating that the expected value of the variable at any time is equal to the current value and variance of the change in the variable in a time interval of T is equal to T.

Based on the standard Brownian motion, the generalized Brownian motion process with drift rate of $\mu$ and variance of $\sigma^2$ is defined as follow (Ross 2007):

(i) $X(0)=0$;

(ii) $\{X(t), t \geq 0\}$ has stationary and independent increments;

(iii) $X(t)$ is normally distributed with mean $\mu t$ and variance $t\sigma^2$.

However, in many cases, the assumption of constant drift rate is not appropriate. Geometric Brownian motion (GBM) is useful for modeling this type of cases. If $\{X(t), t \geq 0\}$ is a Brownian motion process with drift rate of $\mu$ and variance of $\sigma^2$, then Geometric Brownian motion process is defined by:

$$Y(t) = e^{X(t)}$$  (1)

The geometric Brownian motion (GBM) process has gained wide acceptance as a valid model for the growth in the price of a stock over time (Marathe and Ryan 2005). Also, in 2002 Nembhard et al. quantified cost of applying quality control charts using real option pricing methods, where both the sales volume and the price of a product were assumed to follow GBM processes. Thorsen (1998) applied the real options theory to decisions of establishing a new forest stand and assumed that the future net prices of roundwood follow a GBM process. In 2004, Ryan assumed the demand for services in rapidly growing industries follows a GBM and the expansion policy to minimize cost subject to a service level constraint was developed and analyzed.
Removing the seasonality

The first step before testing the assumptions of the GBM is to remove any seasonal variation (Marathe and Ryan 2005). In this paper, by using Multiplicative method (Kendall and Stuart 1983) the seasonality of Asphalt Cement Price Index has been removed.

Figure 2 shows Autocorrelation Function (ACF) plot of the data set indicating that seasonality exists in monthly Asphalt Cement Price Index series. There are two common models for decomposing a time series: additive model and multiplicative model.

Additive model has the form of \( X_t = S_t + T_t + E_t \). That is, the seasonal, trend, and irregular components are added together to give the observed series. In the additive model, the seasonal indices over the periods of a particular season add up to zero. (Brockwell and Davis 2002).

Multiplicative decomposition has the form \( X_t = S_t T_tE_t \). Different from Additive model, the seasonal, trend-cycle and irregular components are multiplied to give the observed series (Makridakis et al. 1998). It has been observed that the seasonal variation of AC monthly price log ratio increases with time. Thus, multiplicative model should be chosen. Figure 3 shows the log ratio of Asphalt Cement Price Index series after removing the seasonality.

![Figure 2. ACF plot of asphalt cement price index](image)
Checking for GBM process fit

There are two assumptions as follow which should be satisfied to assume that the time series follow GBM process (Ross 1999):

1- Normality of the log ratios of $Y(t)$ with constant mean and variance.
2- The log ratios independent of their past values

Normality

There are different tests to check the normality. In this paper Shapiro-Wilk W test is used to check the normality of the log ratios of Asphalt Cement Price Index. In Shapiro-Wilk W test, the hypothesis set is (Shapiro Wilk 1965):

$H_0$: The distribution is normal
$H_1$: The distribution is not normal

The results of the Shapiro-Wilk W show that historical Asphalt cement Price Index data are consistent with the normality assumption of GBM process. W and p-value of the test are 0.9806 and 0.3796 respectively. Since p-value is larger than significance level of 5%, so we cannot reject the null hypothesis which assumes that the time series follows the normal distribution.

Independence from the Past Data

To check that whether the log ratios of $Y(t)$ are independent from their past values, Chi-Square test is used in this paper. In Chi-Square test the hypothesis set is as follow (Blair 1952):

$H_0$: There is no association between the variables

Figure 3. Log ratio of asphalt cement price index after removing seasonality
H₀: Some association does exist.

The results of the Chi-Square show that historical Asphalt Index data are consistent with the memoryless property of GBM process. X-squared, df and p-value are 4.639, 2 and 0.09832 respectively. Since p-value is larger than significant value so we cannot reject the null hypothesis which assume that the data is independent.

**Fit the Model**

The parameters (drift and variance) of the GBM have been estimated using Maximum Likelihood Estimation method. The results show that sample mean and standard deviation are 0.009 and 0.065 respectively. Now, since \( Y_t \) (monthly asphalt cement price index) follows a GBM process, following formula can be used to estimate the index (Luenberger 1995):

\[
E[\ln Y_t] = E[\ln Y_t] + \left( \mu + \frac{\sigma^2}{2} \right) (t - 1) \quad (2)
\]

Monte Carlo Simulation was used to create multiple paths based on the fitted GBM. Figure 4 represents the predict interval of the last year in the data set (June 2011 to June 2012) using simulated paths created by Monte Carlo Simulation.

![Figure 4. Predicted interval for asphalt cement index](image)

**CONCLUSION**

Significant volatility in the price of asphalt cement is one of most critical problems for accurate cost estimation in transportation projects. The ability to model trends and variations in asphalt cement price can help owner organization and contractors to handle the risk of price volatility appropriately. Appropriate stochastic
process can model the volatility of the price index properly. Basic assumptions of GBM are tested in order to fit an appropriate GBM to the dataset. The results of the Shapiro-Wilk test showed that historical asphalt cement price index data are consistent with the normality assumption required by the GBM process. The results of the Chi-Square showed that historical prices are consistent with the memoryless property of GBM process. Followed by approval of these basic assumptions a GBM stochastic process was fitted and its parameters were estimated. Based on the results of the GBM estimations, Monte Carlo simulation was used to simulate the future random paths for the Asphalt Cement Index. The future random paths can be used by contractors and State DOTs to achieve better cost estimations, more accurate budget planning and risk management strategies in highway transportation projects.

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