Enabling 3D Topological Analysis for Building Models with Boundary Representation

Xing SU¹ and Hubo CAI²

¹Xing Su, Ph.D. LEED AP, Department of, School of Engineering, Southern Illinois University at Edwardsville, Edwardsville, IL 62025; PH (618)-650-5899; email: xsu@siue.edu
²Hubo Cai, Ph.D., P.E., GISP, Assistant Professor, Division of Construction Engineering and Management, School of Civil Engineering, Purdue University, 550 Stadium Mall Drive, West Lafayette, IN 47906; email: hubocai@purdue.edu

ABSTRACT

3D topological analysis for building models can remarkably facilitate construction planning in detecting construction activity interferences and deriving partial building models based on topological and locational constraints to meet the demands from team collaboration and task distribution. A widely used 3D topology framework is the 9-Intersection-Model (9-IM) that describes two spatial objects’ topological relationship by identifying intersects between their exterior, boundary, and interior. However, some existing building models using Boundary representation (B-rep) (i.e. CAD) do not naturally carry information about each 3D object’s exterior, boundary, and interior and thus, challenge arises in topological analysis for building models. Most collision detection algorithms only detect whether two 3D objects intersect with each other, but do not support the analysis of detailed topological relationships. This study presents a Vertex Examination Method (VEM) that enables a full range of 3D topological analyses for building models with boundary representation, using the Minkowski-sum calculation. Each vertex of a 3D object is examined by conducting Minkowski-sum with the volume of another 3D object to identify their vertex-volume relationship. The topological relationship between two 3D objects can be determined based on the vertex-volume relationship pattern. This method complements the 3D collision detection by enabling 3D topological analysis between any 3D objects with boundary representation. It is expected to benefits many construction planning tasks, such as site layout, workspace management, and scheduling, by facilitating analytical reasoning and operation on building models. At last, the method limitations and the potential to extend to 4D topological analysis are discussed.

INTRODUCTION

3D/4D construction models have been widely used in many construction projects. Hartmann et al. (2008) summarized seven application areas of 3D/4D construction models based on case studies, including photorealistic renderings, visual design review, analyzing design options/building operations, cost estimating, analyzing construction operations, construction document production, and bid package preparation. Among them, analyzing construction operations via 4D animations is the most popular application. 4D animation is capable of displaying the construction progress at a particular time, simulate the construction for certain time
intervals, or virtually step through construction schedules (Song and Chua 2006, Hartmann and Fischer 2007).

The 4D animation technique significantly facilitates construction planning in analyzing operations. Koo et al. (1998) demonstrated that 4D animation could assist in identifying problems that may normally be overlooked in a traditional schedule representation. Mahalingama et al. (2010) highlighted the advantages of communicating construction plans and processes to clients, visually comparing constructability of work methods to detect conflicts or clashes, and reviewing project plans through animations.

However, the analysis process via 4D animation still relies on personal experience and expertise to identify construction problems. The construction model itself has none or very limited analytical capabilities. The lack of such analytical capabilities places heavy burden of construction operation analysis on the shoulder of construction planners, and human judgment is often error prone. Construction planning demands a construction model that enables analytical capabilities, and an essential part of construction analysis is the topological analysis.

Topology is one of the mechanisms to describe relationships between objects. For two spatial objects, their spatial topological relationship indicates a comparative location between each other, such as inside, overlaps, disjoints, and so on. Construction topology describes spatial relationships between site objects. A construction site is complex that activities occupy different spaces and interact with each other. Some typical construction activity interactions include workspace conflicts, interactive crane workspace, and hazardous areas overlay.

The interactions can be systematically formatted into topological relationships to facilitate automatic detection based on the construction model. For example, from the construction topological perspective, a construction activity that damages an uncured concrete slab can be attributed to an improper planning that the workspace of the activity overlaps the concrete slab during curing; an overlap of workspace may result in a workspace conflict; if a crane’s working range does not cover all desired space in the scheduled period, the crane is not positioned appropriately. A site hazardous situation may be attributed to an improper arrangement that a labor workspace overlaps a hazardous area. In summary, construction topology supports topological analyses on spatial relationships between site activities and helps explanatively communicate the construction plans and processes to clients.

Besides detecting construction activity interference, construction topological analysis can be used for deriving partial building models based on topological and locational constraints. As central model management servers are emerging as the most important IT infrastructure component nowadays, highly collaborative design and engineering of buildings become possible and highly demanded; and a very important prerequisite for asynchronous collaborative design and engineering is the creation of partial models (Borrmann & Rank 2009). Construction topology plays a critical role of being the theoretical foundation to support queries and reasoning to derive partial models.
EXISTING 3D TOPOLOGY FRAMEWORKS

Many researches have been carried on in GIS area about 3D spatial relations (De La Losa and Cervelle 1999, Coors 2003, Breunig and Zlatanova 2005, Penninga and Van Oosterom 2007). The three main spatial relations include metric (distance, closerThan, furtherThan, etc.), directional (strict/relax above, northOf, etc.), and topological (meet, cover, overlap, etc.), among which topology has been identified as “a central and important concept in GIS since mid-1970s.” (Van Oosterom et al. 2006). In a 3D construction context, topological relationships can be used to answer questions such as “find any sewage pipes and cables that intersect with the trench to be cut in this area”, “find the shortest route for the equipment’s transportation on site”, and “find an appropriate location for the crane to cover the building panel installation work.”

The first substantial progress towards a formalization of topological relationships was the development of the 4-Intersection Model (4-IM) by Egenhofer and Herring (1990) and Egenhofer and Franzosa (1991). The model forms a 2x2 matrix to specify the semantics of topological predicates between two objects by determining the intersections between the interior and the boundary of the two objects. Theoretically there are $2^4$ possible configurations but in reality only a subset could happen. The concept was first employed in 1D space and later extended to 2D space (Egenhofer and Franzosa 1991). The 4-IM was extended to 9-Intersection Model (9-IM) by incorporating the exteriors to solve topological relations between elements including lines and regions. Similarly, the 9-IM examines the intersections of each of the interior, boundary, and exterior of object A with each of the interior, boundary, and exterior of object B, resulting in the 3x3 matrix as follows:

$$I = \begin{bmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^\circ \cap B^- \\ \partial A \cap B^\circ & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^\circ & A^- \cap \partial B & A^- \cap B^- \end{bmatrix}$$

where $A^\circ$ denotes the interior, $\partial A$ denotes the boundary, $A^-$ denotes the exterior of A, and $\cap$ represents the intersection operator. Each element is a binary value resulting from the intersecting operation. It gives a total of 512 possible configurations although many of them are not achievable in practice. This framework is updated to support 3D objects by Zlatanova(2000). There are 8 basic possible relationships between 3D objects, which are illustrated in Figure 1.
Figure 1. 9-IM configurations

Despite some criticism, i.e. some intuitively different topological configurations result in the same 9-IM matrix (Borrman & Rank 2009), this framework provides a systematic and easy-to-implement method of detecting spatial relationships (Zlatanova et al. 2004). It has been further developed and widely used by a number of researches in different areas (Egenhofer and Franzosa 1994, Chen et al. 2001, Papadias and Theodoridis 1997).

**KNOWLEDGE GAP**

Borrmann and Rank (2009) developed an Octree-based method to enable topological analysis of 3D building models in the IFC format. A knowledge gap still exists in construction topology for building models with Boundary representation (B-rep). In construction, CAD and GIS often use B-rep as their geometric data format.

B-rep represents a solid or closed object as a collection of connected surface elements. The inside of the closed surface elements is considered as solid and the outside as non-solid. B-rep models usually include two parts: geometry and topology. The geometry carries the shape and position information and the topology indicates spatial relationships between the geometries. Typical geometry elements include surfaces, curves, and points. The main topological items are faces, edges, and vertices. A face is a bounded piece of surface and comprises multiple edges. An edge is a bounded piece of a curve and comprises two vertices. A vertex lies at a point. Figure 1 shows a hierarchical data structure of Multi-patch, a typical B-rep format widely used in GIS, to illustrate the concept. The “TriangleStrip”, “TriangleFan”, “Triangles”, and “Ring” are different types of faces.
However, it takes a lot of effort to determine a topological relationship between two 3D geometries based on the 9-IM by examining whether there are geometrical intersections in every pair of combination among the interior, boundary, and exterior. For example, A “overlaps” B only when they meet the requirement of a specific intersection pattern

\[
\begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
0 & 1 & 1
\end{bmatrix}
\]

by checking \[A \cap B^+ \quad A \cap \partial B \quad A \cap B^-\];

\[
\begin{bmatrix}
0 & 0 & 1
\end{bmatrix}
\]

similarly, C is called “covers” D only when they meet the requirement of a specific intersection pattern

\[
\begin{bmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
0 & 1 & 1
\end{bmatrix}
\]

by checking \[C \cap D^+ \quad C \cap \partial D \quad C \cap D^-\].

Theoretically, it takes nine steps to determine a topological relationship by examining each element of the matrix. It is especially a challenge for B-rep to adopt 9-IM because B-rep does not naturally support the 9-IM examination. B-rep represents each 3D shape by a boundary or shell but does not directly carry information about interior and exterior.

**MINKOWSKI SUM FOR SPACE CONFLICT DETECTION**

The major progress towards a construction topology framework from previous studies is the development of space conflict detection methods. Akinci et al. (2002c) employed discrete event-simulation mechanisms for space time conflict detection, which first identifies concurrent activities and then checks possible geometric clashes for each pair of candidate concurrent activities. Guo (2002) defined space conflict as more than one space demand claim on a specific available space during the same time period. Chavada et al. (2011) proposed a conflict detection process that examines both the temporal conflicts between schedule and the spatial conflict between workspaces.

For all these methods, 3D spaces must be examined to detect whether there are any intersects. A great amount of researches have been done for boundary
representation models to check whether two 3D objects collide, also known as 3D collision detection. Some widely used algorithms include separating axis theorem (SAT) or hyperplane separation theorem, Gilbert–Johnson–Keerthi algorithm (GJK), and Expanding Polytope Algorithm (EPA). Those algorithms are well developed based on the Minkowski sum method. For two point sets A and B, their Minkowski difference is defined as:

\[ A \oplus (-B) = \{a + (-b) : a \in A, b \in B\}, \]

where \( a + (-b) \) is the vector sum of the position vectors a and (-b). A 2D example in Figure 3.2 illustrates the concept of Minkowski difference. Square A has vertices of (2,0), (4,0), (4,2), (2,2); and triangle B has vertices of (1,1), (3,1), and (1,3). Their Minkowski difference is \{(1,-1), (-1,-1), (1,-3), (1,-1), (3,-3), (3,1), (1,1), (3,-1), (1,1), (-1,1), (1,-1)\}. The fact that the origin is inside \( A \oplus (-B) \) indicates A and B intersect. Further, if the origin lies on the boundary of \( A \oplus (-B) \), A meets B; and if the origin is outside \( A \oplus (-B) \), A disjoints B. The concept remains valid when it extends to 3D.

\[ \text{Figure 3. A Minkowski Sum Illustration} \]

**TOPOLOGICAL AMBIGUITY**

From a topological perspective based on 9-IM, if A and B intersect, there are six possibilities: overlaps, contains, covers, inside, covered by, and equals. The six topological relationships cannot be distinguished from each other by the Minkowski sum result, which is referred as topological ambiguity in this paper. As shown in Figure 4, the topological relationships “Covers”, “Covered by”, “Contains”, “Inside” result in the same result that the origin lies within the Minkowski sum.
A VERTEX EXAMINATION METHOD

A Vertex Examination Method (VEM) is proposed here to distinguish the six topological relationships. VEM examines whether a 3D object’s vertices are inside, on, or outside another 3D object. Different topological relationships result in different patterns of Vertex/Volume (V/V) relationships. Table 1 describes the determinants in plain words.

Table 1. Criteria to determine some topological relationships

<table>
<thead>
<tr>
<th>Topological Relationship</th>
<th>V/V Relationship Pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>A contains B</td>
<td>All A’s vertices are outside B &amp; All B’s vertices are inside A</td>
</tr>
<tr>
<td>A inside B</td>
<td>All A’s vertices are inside B &amp; All B’s vertices are outside A</td>
</tr>
<tr>
<td>A equals B</td>
<td>All A’s vertices are on B &amp; All B’s vertices are on A</td>
</tr>
<tr>
<td>A covers B</td>
<td>There is at least 1 A’s vertex outside B</td>
</tr>
<tr>
<td></td>
<td>There is no A’s vertex inside B</td>
</tr>
<tr>
<td></td>
<td>There is at least 1 B’s vertex on A</td>
</tr>
<tr>
<td></td>
<td>There is no B’s vertex outside A</td>
</tr>
<tr>
<td>A covered by B</td>
<td>There is at least 1 A’s vertex on B</td>
</tr>
<tr>
<td></td>
<td>There is no A’s vertex outside B</td>
</tr>
<tr>
<td></td>
<td>There is at least 1 B’s vertex outside A</td>
</tr>
<tr>
<td></td>
<td>There is no B’s vertex inside A</td>
</tr>
<tr>
<td>A overlaps B</td>
<td>Else</td>
</tr>
</tbody>
</table>

The V/V relationship pattern can be represented by a 2x3 matrix as shown in Table 2. Each element of the matrix gives information whether the set that meets a specific V/V relationship is empty (“0”), not empty (“1”), or having no effect (“*”).
For example, if there are A’s vertices inside B’s volume, the left top cell will have a value “1”. If there is no B’s vertex outside A’s volume, the right bottom cell will have a value “0”. The wildcard “*” is used at certain places in the matrix indicating that is not relevant for the particular predicate. Referring to “A covers B” in Table 1 as an example, the predicate matrix for this topological relationship will be 
\[
\begin{bmatrix}
0 & * & 1 \\
* & 1 & 0
\end{bmatrix}
\]
which means it can be determined that A covers B as long as there is at least 1 A’s vertex outside B, no A’s vertex inside B, at least 1 B’s vertex on A, and no B’s vertex outside A. Whether there is A’s vertex on B or B’s vertex inside B does not affect the result.

<table>
<thead>
<tr>
<th>Table 2. The V/V relationship matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inside</td>
</tr>
<tr>
<td>VerA vs. VolB</td>
</tr>
<tr>
<td>VerB vs. VolA</td>
</tr>
</tbody>
</table>

Table 3 displays the translating result from the plain words in Table 3.1 to the V/V relationship matrix. All the criteria in table 1 are translated into a matrix format. Each V/V relationship matrix determines a specific topological relationship.

<table>
<thead>
<tr>
<th>Table 3. V/V relationship matrices to determine topological relationships</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topological Relationship</td>
</tr>
<tr>
<td>--------------------------</td>
</tr>
</tbody>
</table>
| A contains B             | \[
\begin{bmatrix}
0 & 0 & 1 \\
1 & 0 & 0
\end{bmatrix}
\] |
| A inside B               | \[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\] |
| A equals B               | \[
\begin{bmatrix}
0 & 1 & 0 \\
0 & 1 & 0
\end{bmatrix}
\] |
| A covers B               | \[
\begin{bmatrix}
0 & * & 1 \\
* & 1 & 0
\end{bmatrix}
\] |
| A covered by B           | \[
\begin{bmatrix}
-1 & 0 \\
0 & -1
\end{bmatrix}
\] |
| A overlaps B             | Else |

All the above mentioned algorithms are for convex shapes. For computing Minkowski sum between general polyhedrons (including concave shapes), there are two main methods: divide-and-conquer and convolution, but both are hard to implement in 3D. Even though some efforts have been attempted to address the problem (Ericson 2004, Lien 2009, Hsieh et al. 2010), the 3D concave collision detection is still considered an open research area. In this study, 3D concave collision...
detection is not supported. Users have to manually decompose the concave shapes; otherwise the bounding box of each concave shape will be considered for topological relationship detection.

CONCLUSION AND DISCUSSION

This study presents a Vertex Examination Method (VEM) that enables a full range of 3D topological analyses for building models with B-rep format, using the Minkowski-sum calculation. With the V/V relationship matrices that represent the V/V relationship patterns by matrices, this method is ready to be implemented for computer-aided topological relationship detection. VEM bridges the gap of 3D topological analysis for building models with B-rep format and serve as a theoretical foundation for future extension of a construction 4D topology framework.

Construction 4D topology extends the spatial topology into a 3D + time environment, which is able to describe a dynamic spatial-temporal relationship between two construction activities. Construction 4D topology should provide a formal approach to modeling and capturing spatial temporal relationships between construction activities and thus, is beneficial to the analysis of construction site dynamics interactions. A well-defined construction 4D topology framework should be able to describe and analyze not only the pre-defined interactions such as workspace conflicts and crane layout requirements, but also any possible spatial temporal relationships occur on site.

REFERENCE


